

## LOW-MASS WHITE DWARFS AND THE COOLING SEQUENCES IN THE HYADES CLUSTER

CHAO-WEN CHIN AND RICHARD STOTHERS

Goddard Institute for Space Studies, NASA, New York

*Received 1970 April 20; revised 1970 August 24*

### ABSTRACT

Detailed evolutionary sequences have been constructed for models of white dwarfs with very low mass (0.1, 0.2, and 0.3  $M_{\odot}$ ) and with a variety of chemical compositions. Convection near the surface is taken fully into account in the models with hydrogen envelopes. Down to a luminosity of at least  $\log (L/L_{\odot}) = -3.3$ , the basic properties of the models are not affected significantly by the presence of convection in the hydrogen-rich layers. All the models with envelopes assumed to be in radiative equilibrium are insensitive to the choice of chemical composition of the envelope, at least as long as the mass of the envelope encompasses 1 percent, or less, of the total mass.

The color-magnitude diagram of the white dwarfs in the Hyades has been reinterpreted. Two major mass sequences are isolated: a "blue" sequence at  $\sim 1 M_{\odot}$  and a probable "red" sequence at  $\sim 0.1 M_{\odot}$ , with a scatter of intermediate-mass stars. The observed luminosity functions seem to be in good agreement with the theoretical ones. If the red sequence is real, then the bifurcation of the white-dwarf masses occurs at a parent mass of  $2.4 \pm 0.4 M_{\odot}$ . A speculative picture is presented of the origin of the white-dwarf sequences.

### I. INTRODUCTION

The origin and mass frequencies of white dwarfs are still imperfectly known. Since the white dwarfs in the general field form an undifferentiated sample of stars with different origins, masses, and ages, it is useful to look to members of well-observed clusters for clarification. In this paper, the subluminous members of the Hyades will be investigated in some detail.

Since adequate models for cooling white dwarfs of high and intermediate mass have already been constructed by a large number of workers, we shall concern ourselves here first with computing detailed evolutionary sequences and lifetimes for white dwarfs of low mass with a variety of chemical compositions for the envelope and core. In the case of hydrogen envelopes, convection near the surface will be taken into consideration. Uncertainties in the models, which arise mainly from atomic opacities and ion condensation effects, will be assessed against uncertainties imposed by observational limitations.

Earlier sequences of low-mass models have been constructed by Kaminisi (1956), Vila (1967), and Kippenhahn, Thomas, and Weigert (1968). However, in each of these references, only one low-mass sequence was presented, convection was ignored, and no mass smaller than 0.2  $M_{\odot}$  was selected. It will be shown here that a very small mass yields a somewhat different picture of evolution.

### II. BASIC STRUCTURE AND ASSUMPTIONS

In this paper, we consider only those evolutionary stages of stars after the completion of nuclear burning. Having lost most of their original envelope, the stars are evolving toward lower luminosity, drawing their energy from gravitational contraction and from thermal cooling of the electrons and ions. Masses of 0.1, 0.2, and 0.3  $M_{\odot}$  will be considered here. As a choice of chemical composition, we have adopted one of the following for the envelopes:

H: Hydrogen-rich mixture (Cameron I):  $X = 0.739$ ,  $Y = 0.240$ ,  $Z = 0.021$ .

He: Helium-rich mixture (Weigert I):  $X = 0$ ,  $Y = 0.956$ ,  $Z = 0.044$ .

C/O: Carbon-oxygen mixture (Weigert VI):  $X = 0$ ,  $Y = 0$ ,  $Z_c = 0.4995$ ,  $Z_o = 0.4995$ .

In the above the notation follows that of Cox and Stewart (1965). The cores are composed of either the helium-rich mixture or the carbon-oxygen mixture, and the mass fraction contained in the core,  $q_{\text{core}}$ , is taken to be either 0.99 or 1.0.

In nondegenerate and semidegenerate regions, the full temperature-dependent, non-relativistic equation of state for partially degenerate matter has been used. In highly degenerate regions where the Fermi degeneracy parameter  $\Psi$  exceeds 20, the temperature gradient has been taken to be zero. Salpeter's (1961) corrections to the equation of state are small in all our models; nevertheless, their slight effect has been evaluated by parallel computations for the most degenerate models. The classical Coulomb effect is found to dominate, but all the effects together cause a change of less than 5 percent in central temperature, effective temperature, or lifetime.

Opacities have been taken from the tables of Cox and Stewart (1965). In degenerate domains where values of the opacity are not given, the tables have been supplemented by Hubbard and Lampe's (1968) results. In order to take account of the interior energy release, we have assumed a uniform energy source as a good approximation (see Hayashi, Hōshi, and Sugimoto 1962). The method of solution of the basic equations follows the lines prescribed by Schwarzschild (1958) and Hayashi *et al.* (1962).

### III. CONVECTION IN THE ENVELOPE

The importance of convection in the envelope of cool white dwarfs was first pointed out by Schatzman (1958) and later confirmed by Böhm (1968) in a detailed calculation of the atmosphere of van Maanen 2. Böhm obtained a temperature of  $9.8 \times 10^5$  °K at the bottom of the convective zone for two assumed cases:  $T_e = 5780$  °K,  $g = 10^8$  cm sec<sup>-2</sup>; and  $T_e = 5040$  °K,  $g = 3.16 \times 10^7$  cm sec<sup>-2</sup>. He identified this temperature with the transition temperature  $T_{\text{tr}}$  between the envelope and the (almost) isothermal core. More recently, Van Horn (1970) has obtained envelope temperatures down to the point  $\Psi = 0$  for masses ranging from 0.22 to 1.22  $M_\odot$  and luminosities ranging from  $\log (L/L_\odot) = -2$  to  $-4$ . It should be noted that neither  $T_{\text{tr}}$  nor  $T(\Psi = 0)$  should be identified with the central temperature.

In the case of the hydrogen-rich envelopes of our present models, calculations of both radiative and convective transport have been undertaken for the sake of comparison. The mixing-length theory of convection has been used for the outer layers, as described by Iben (1963), who kindly loaned us his computer program which fixes essentially the adiabatic parameter  $E$  (Schwarzschild 1958). The ratio of mixing length to density scale height has been taken equal to 0.5. Our calculations were carried out without knowledge of Van Horn's (1970) work, but, since his paper has appeared, it seems worthwhile to compare our results with his wherever possible.

The results of our calculations for hydrogen envelopes of low-mass white dwarfs are based on a complete solution of the interior structure of the star, and can be summarized as follows.

1. The radius fraction covered by the convective layers begins to exceed 0.02 when the luminosity drops below  $\log (L/L_\odot) = -4.0$  and  $-2.0$  for 0.3  $M_\odot$  and 0.2  $M_\odot$ , respectively. The whole evolutionary sequence for 0.1  $M_\odot$  is characterized by a deep convective envelope. When  $\log (L/L_\odot) = -3.3$ , convection encompasses a radius fraction of 0.17, 0.03, and 0.015 for masses of 0.1, 0.2, and 0.3  $M_\odot$ , respectively.

2. Convection reduces  $\log T_e$  from its corresponding value in the assumed radiative case by about 0.07 at 0.1  $M_\odot$ , but by less than 0.01 at 0.2  $M_\odot$  and 0.3  $M_\odot$  for luminosities brighter than  $\log (L/L_\odot) = -3.3$ . An improved treatment of the convective envelope would probably lower  $\log T_e$  somewhat more.

3. The base of the convective envelope is only very slightly degenerate in all cases. The Fermi degeneracy parameter at the base of the envelope never exceeds  $\Psi = 0$  for  $\log (L/L_\odot) > -3.3$ .

4. The temperature at the base of the convective zone is, in all our deep-zone models,

$(1-3) \times 10^6$  °K. This result and the similar results of Böhm and Van Horn are in accord with the approximate universality of this value of base temperature in virtually all stellar models with thick convection zones at the surface, as was first pointed out by Osterbrock (1953) in the case of red main-sequence dwarfs and the Sun.

5. On account of the weakness of the envelope degeneracy just mentioned, there exists immediately below the convective layers a significantly radiative and weakly degenerate region where the opacity is still fairly high (see also Böhm 1968, Table II). The interposition of this weakly degenerate radiative zone makes the structure and temperature of the inner "isothermal" core almost independent of the thin outer convective layers. This point has been verified by the parallel computations for models with envelopes assumed to be radiative out to the surface. The same conclusion can be inferred from Van Horn's (1970) Figure 1 and associated text, although, as Van Horn points out, convection may have an important effect for luminosities fainter than  $\log (L/L_\odot) = -3.3$ , where degeneracy becomes important near the bottom of the convective zone. The critical luminosity at which degeneracy appears at the bottom of the convective zone is approximately the same for all masses up to at least  $1.22 M_\odot$  (Van Horn 1970).

6. The central temperature exceeds the temperature at the bottom of the convective zone by factors of 4, 6, and 6 for masses of 0.1, 0.2, and  $0.3 M_\odot$ , respectively, at a luminosity of  $\log (L/L_\odot) = -3.3$ . As a consequence, the lifetime, which is determined by the temperature of the nearly isothermal core, is little affected by the thin convective layers in white dwarfs heavier than  $0.1 M_\odot$  and brighter than  $\log (L/L_\odot) = -3.3$ . Our parallel computations for models with the outer envelope assumed to be radiative to the surface confirm that the lifetimes above  $\log (L/L_\odot) = -3.3$  are changed by less than 3 percent due to the presence of the convective layers.

The effect of convection on complete stellar models with envelopes composed predominantly of helium or of carbon and oxygen has not been investigated here, although Van Horn (1970) has considered the related envelopes.

#### IV. EVOLUTIONARY TRACKS WITH VARIOUS CHEMICAL COMPOSITIONS

Evolutionary tracks on the theoretical H-R diagram are shown in Figure 1 for white dwarfs composed of (a) helium throughout and (b) helium cores and hydrogen envelopes. The inclusion of an envelope whose chemical composition is lighter than that of the core has been found to make little difference unless the envelope composition is hydrogen-rich; in that case the radius is increased somewhat, as Hamada and Salpeter (1961) found for their zero-temperature models. Evolutionary tracks for the other choices of chemical composition are nearly identical with those for the homogeneous helium models, except in the upper region of the H-R diagram where the evolutionary tracks show a sharp bend, as exhibited in the selected case of  $0.1 M_\odot$ . Our sequence for  $0.2 M_\odot$  with a carbon-oxygen core and a helium envelope is in excellent agreement with Vila's (1967) analogous sequence if a very slight adjustment of the total radius is made to allow for the smaller mass contained in the envelope of Vila's models. Similarly good agreement is found by interpolation with the sequence for  $0.26 M_\odot$  containing a helium core and a hydrogen envelope which was calculated by Kippenhahn *et al.* (1968).

The lifetimes of the various model sequences are remarkably similar. This is true despite the variations in mass and in chemical composition of the envelope and core. On the supposition that thermal cooling of the ions is the only energy source, a white dwarf of high mass should have a longer cooling time at a given luminosity level than a white dwarf of low mass (Schwarzschild 1958). But the latter object is, in reality, less degenerate and therefore releases more energy by gravitational contraction at the luminosities under consideration in this paper. An analytic formula for the cooling time, derived, for example, by Schwarzschild (1958), is more accurate when the degeneracy is greater (higher mass or lower luminosity). However, the analytic formula does indicate

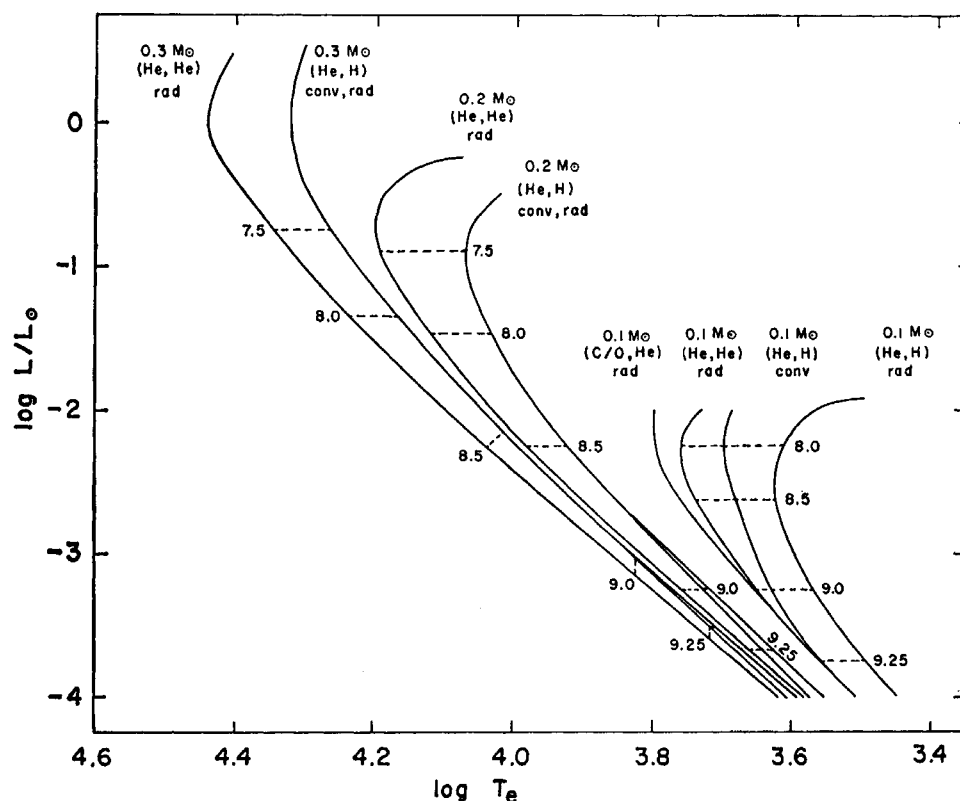


FIG. 1.—Theoretical H-R diagram for the evolutionary sequences of low-mass white dwarfs. The tracks are labeled (in sequence) with the mass, chemical composition of core and envelope, and surface condition. For 0.2 and 0.3  $M_{\odot}$ , the tracks for C/O cores are virtually identical with those for He cores except at high luminosity. Loci of constant time are labeled with  $\log \tau$  (years) and refer to He cores only; ages for C/O cores are approximately 30 percent smaller.

qualitatively why the changes in envelope composition have only a small effect on the cooling time below the evolutionary bend in the H-R diagram. Moreover, it shows that the core composition enters the cooling time as the inverse of the mean molecular weight of the ions: for our carbon-oxygen and helium core compositions, the ratio of cooling times is expected to be 14/4, whereas our detailed computations yield a ratio of only  $\sim 1.3$ , owing to the effects of incomplete degeneracy.

One complicating factor has not been included in our detailed models, however. As Mestel and Ruderman (1967) and others have pointed out (see the recent discussion by Greenstein 1969), the specific heat of ions in the phases of ion liquefaction and crystallization may be twice as large as the classical value within the approximate temperature range

$$4 \times 10^3 \rho^{1/2} < T(^{\circ} \text{K}) < 1 \times 10^5 Z^{5/3} \rho^{1/3}.$$

The increase in specific heat of the ions slows down the cooling of the white dwarf.

Ion liquefaction begins to set in at the stellar center when the luminosity is still very high if the core is composed of carbon and oxygen, but only when  $\log (L/L_{\odot}) = -3.3$ ,  $-2.2$ , and  $-1.8$  (for 0.1, 0.2, and 0.3  $M_{\odot}$ , respectively) if the core is composed of helium. Crystallization is complete at the center when  $\log (L/L_{\odot}) < -4$ ; thereafter the white dwarf cools off very rapidly. In low-mass white dwarfs, however, much of the energy release comes from gravitational contraction and thermal cooling of the electrons.

Unpublished computations by Vila (1969) show that the cooling time of carbon-oxygen models at a luminosity of  $\log (L/L_{\odot}) = -3.3$  is increased by only  $\sim 10$  and  $\sim 30$  percent for masses of  $0.2$  and  $0.4 M_{\odot}$ , respectively. For a mass of  $0.1 M_{\odot}$  or for helium cores, the percentage increase would be much smaller.

Because of ion condensation effects in the core and deep convection in the envelope, our model computations for luminosities fainter than  $\log (L/L_{\odot}) = -3.3$  are not very secure. However, the uncertainties in the present opacities are probably unimportant, because our calculated cooling times are found to be rather insensitive to changes in the chemical composition of the envelope. Fortunately, the investigation of white dwarfs in the Hyades which we shall make requires only the following: luminosities down to  $\log (L/L_{\odot}) \approx -3$ , rough accuracy in the effective temperatures, and an accuracy of, say, 50 percent in the lifetimes. It seems fairly certain that our models can provide this sort of accuracy.

Table 1 presents details of two model sequences for a mass of  $0.1 M_{\odot}$ . A subscript  $a$  denotes the base of the convective envelope. The carbon-oxygen sequence is included in order to extend the network of models presented by Vila (1966, 1967).

#### V. WHITE DWARFS IN THE HYADES

In the Hyades Cluster, the basic data on stellar statistics are probably very nearly complete except at the lower end of the main sequence (van Altena 1966, 1969; Eggen 1969). Eggen lists twenty-one white dwarfs (if we omit one faint outlying object), in excellent agreement with Sandage's (1957) expectation of about twenty-three dead stars on the basis of an extrapolation of the observed luminosity function for the main sequence. In this paper we shall assume that all twenty-one objects are true members of the Hyades, chiefly on the basis of van Altena's proper-motion criteria. In addition, the blue objects have as corroborative evidence of their membership the rarity of similar stars of appropriate magnitude and color. Eggen (1968) previously expressed some doubt about membership of several of the red objects, but apparently he now considers them to be probable members (Eggen 1969), as does van Altena (1969). Eggen's absolute magnitudes have been adopted here. A possible error of up to  $\pm 0.4$  mag in the individual

TABLE 1  
SELECTED EVOLUTIONARY MODELS FOR WHITE DWARFS OF  $0.1 M_{\odot}$  ( $q_{\text{core}} = 0.99$ )

Chemical Composition	$\log (L/L_{\odot})$	$\log T_e$	$\log T_c$	$\tau$ ( $10^9$ years)	$\Psi_a$	$\log T_a$	$r_a/R$
He core, H envelope (convective)	-2.00	3.71	7.19	0	-3.1	6.34	0.66
	-2.25	3.71	7.20	0.093	-2.9	6.28	0.74
	-2.50	3.69	7.16	0.22	-2.7	6.25	0.79
	-2.75	3.67	7.09	0.38	-2.3	6.26	0.83
	-3.00	3.65	7.02	0.61	-1.4	6.30	0.83
	-3.25	3.63	6.93	0.98	-0.2	6.38	0.83
	-3.50	3.60	6.84	1.5	+1.5	6.44	0.83
	-3.75	3.57	6.68	2.1	+4.1	6.42	0.83
C/O core, He envelope (radiative)	-2.00	3.81	7.36	0	...	...	...
	-2.25	3.80	7.35	0.072	...	...	...
	-2.50	3.77	7.31	0.17	...	...	...
	-2.75	3.74	7.25	0.30	...	...	...
	-3.00	3.70	7.18	0.49	...	...	...
	-3.25	3.66	7.11	0.76	...	...	...
	-3.50	3.61	7.01	1.2	...	...	...
	-3.75	3.56	6.90	1.7	...	...	...

absolute magnitudes is caused by uncertainty of the zero point of the parallax and by dispersion in the distances of individual cluster members (see also the discussion by van Altena 1969); however, an error of even 0.4 mag is unimportant for our purposes here or in Stothers (1970).

Historically, two parallel sequences of white dwarfs have been distinguished on an observational ( $M_v$ ,  $U - V$ )-plot for all white dwarfs with well-determined distances (Eggen and Greenstein 1965; Eggen 1969). The white dwarfs in the Hyades appear to populate both of these sequences. But a much better description of the white dwarfs in the Hyades is a subdivision of them into two color groups, each of which forms a separate sequence (cooling curve) based on mass. One group contains ten blue white dwarfs of  $\sim 1 M_\odot$ , and the other group contains eleven red white dwarfs (and subdwarfs) of  $\sim 0.1 M_\odot$  (see Fig. 2). No explanation can otherwise be advanced for the presence of the red subluminal stars in the Hyades. Nevertheless, it is easily seen from Figure 2 how one or two apparent (but spurious) sequences can arise if one connects the "blue" points near  $\log (L/L_\odot) = -2$ ,  $\log T_e = 4.4$  with the "red" points near  $\log (L/L_\odot) = -3$ ,  $\log T_e = 3.7$ . We do not venture to say whether the white dwarfs in the general field will be similarly resolved; the available data (Eggen 1969; Greenstein 1969) seem to be inadequate to answer this question at present.

In what follows we shall assume the reality of the red sequence in the Hyades. We have considered several possible interpretations on the basis of this assumption, but the particular picture which emerges below is the one we feel to be most self-consistent at the present time. We wish to emphasize the tentative and speculative nature of our explanation for the presence of two white-dwarf sequences in the Hyades and the necessity of further observational work to establish or rule out the existence of the red one.

The luminosity function for the ten blue white dwarfs in the Hyades has been tabulated and discussed by Stothers (1970). It is in good accord with the theoretically expected luminosity function based on published stellar models (Vila 1966, 1967) and the age of the Hyades as deduced from theoretical fitting of the main sequence,  $\log \tau$

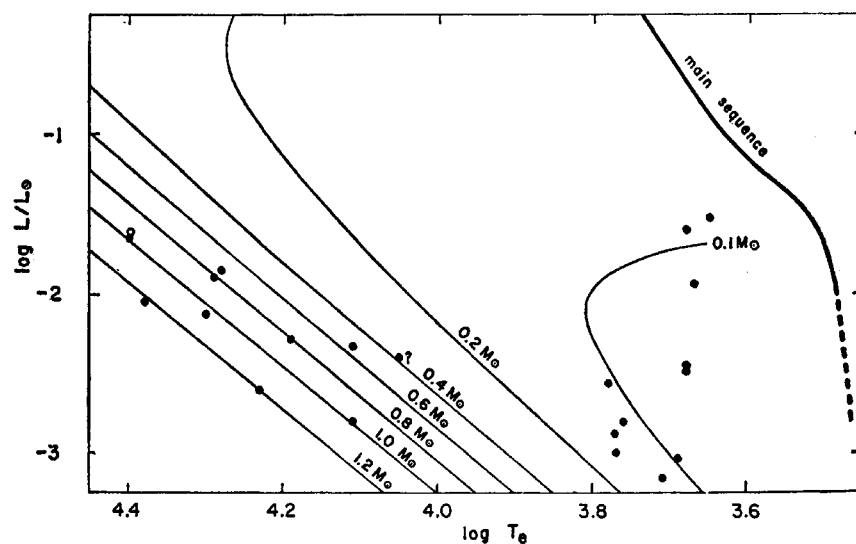


FIG. 2.—Bolometric H-R diagram for subluminal stars in the Hyades (filled circles) and Pleiades (open circle), according to the available published data (Eggen and Greenstein 1965; Eggen 1969) and the transformations mentioned in the text. The evolutionary sequences have been calculated for carbon-oxygen cores and thin helium envelopes (Vila 1966, 1967; this paper). The white dwarf with a question mark may be a composite object.

(years) =  $8.7 \pm 0.1$  (see references in Stothers 1970).<sup>1</sup> By the use of Van Horn's (1968) procedure to convert the observed  $U - V$  colors into effective temperatures and Chandrasekhar's (1939) theoretical mass-radius relation, the masses of the blue white dwarfs turn out to be  $1.0 \pm 0.2 \mathcal{M}_{\odot}$ . By the use of a strictly blackbody interpretation of the observed  $U - V$  colors, the masses are somewhat smaller,  $0.7 \pm 0.3 \mathcal{M}_{\odot}$ . In this paper, we shall adopt  $\sim 1 \mathcal{M}_{\odot}$  as the characteristic mass of the blue white dwarfs, pointing out that the precise value of the mass is not important for the arguments here or in Stothers (1970).

The observational data for the eleven red subluminous stars have been transformed into theoretical quantities by adopting Johnson's (1966) empirical relations for red main-sequence stars with the same  $U - V$  color. On the H-R diagram (Fig. 2), the observed sequence of red subluminous stars appears to follow closely the theoretical sequence for contracting white dwarfs of  $\sim 0.1 \mathcal{M}_{\odot}$  if one allows for uncertainties in the conversion of observational quantities, in a possible slight spread of stellar masses, and in the uncertain chemical composition of envelope and core. The cooling time of the faintest red white dwarf is  $\log \tau$  (years) =  $8.8 \pm 0.1$ , where the error estimate reflects the uncertainties just mentioned. This cooling time is in good agreement with the cooling time of the faintest blue white dwarf and with the main-sequence age of the Hyades.

It is rather unlikely that the stars on the red sequence are pre-main-sequence objects of low mass which have failed to ignite hydrogen and are turning into black dwarfs, as suggested by Eggen (1969). The Hyades Cluster shows a clear continuation of the "normal" main sequence down to the limit of observability (van Altena 1969); it is indicated in our Figure 2. This part of the main sequence in the Hyades agrees very well with its counterpart in the general field (Eggen 1969) and with the main sequence predicted by theoretical stellar models, as shown by a comparison of van Altena's (1969) or Pesch's (1968) empirical main sequence with Ezer and Cameron's (1967) theoretical one. It is well known that the theoretical main sequence is formed, below  $\sim 0.1 \mathcal{M}_{\odot}$ , of stars which do not ignite hydrogen and simply cool off. However, the observed subluminous sequence at  $\sim 0.1 \mathcal{M}_{\odot}$  in the Hyades lies distinctly to the blue of this simultaneously observed, true main sequence. The difference in radius between the two sequences (a factor of about 2) can be ascribed to the heavier core composition of stars on the subluminous sequence if they are the remnants of previous nuclear evolution.

The observed and predicted luminosity functions for the red subluminous Hyades are given in Table 2. Agreement is again good, despite the uncertainty in mass, except for two very bright subdwarfs. At the present time, it is unclear how to account for these two objects, except to suggest here that they might be in a final stage of nuclear burning in a shell near the stellar surface, which would temporarily delay their evolution into the red "cooling" sequence. In fact, such an interpretation may be necessary if one is to explain the simultaneity of the two sequences (see below).

From an evolutionary point of view, the chief difficulty in interpreting the Hyades white dwarfs is the coexistence of two distinct mass sequences, each containing young and old white dwarfs. If these sequences reflect continuous, but nonoverlapping, ranges of initial mass on the main sequence, then the dividing point  $\mathcal{M}^*$  on the main sequence is expected to be close to the initial mass  $\mathcal{M}_{\text{RG}}$  of the present red giants. Assuming that the "universal" normalized luminosity function for main-sequence stars,  $\phi_{\text{MS}}(M_v)$

<sup>1</sup> Van den Heuvel (1970) has recently suggested that the main-sequence age of the Hyades may be as old as  $\log \tau$  (years) = 8.9. This is interesting because white-dwarf models which include ion condensation effects (Vila 1969) are systematically older by about the required amount than the published models which do not include these effects. Therefore, we have decided that the absolute luminosity functions tabulated by Stothers (1970) should probably remain unchanged. In any case, the *relative* luminosity functions certainly remain unchanged.

TABLE 2  
LUMINOSITY FUNCTION FOR RED SUBLUMINOUS STARS IN THE HYADES

log ( $L/L_{\odot}$ )	OBSERVED NUMBER*	PREDICTED NUMBER†	
		0.1 $M_{\odot}$	0.2 $M_{\odot}$
-1.2 to -1.7.....	2	‡	0.8
-1.7 to -2.2.....	1	0.7	2.0
-2.2 to -2.7.....	3	2.4	3.6
-2.7 to -3.2.....	5	5.0	5.0
< -3.2.....	0	0	0

\* Johnson's (1966) transformations are used to obtain log ( $L/L_{\odot}$ ). The table is not significantly different if strictly blackbody transformations are used.

† Insensitive to chemical composition; normalized to the penultimate entry under Observed Number.

‡ Not calculated.

(Sandage 1957), applies to the now-evolved portion of the Hyades main sequence, we have

$$\int_{M_{\text{RG}}}^{M^*} \phi_{\text{MS}}(M_v) \frac{dM_v}{dM} dM = \int_{M^*}^{\infty} \phi_{\text{MS}}(M_v) \frac{dM_v}{dM} dM,$$

since the observed numbers of completely evolved stars (white dwarfs) with presently low and high masses are approximately equal. The solution of this equation for the Hyades is  $M^* - M_{\text{RG}} = 0.4 M_{\odot}$ . For  $X_e = 0.6-0.7$ , we have  $M^* = 2.4-2.8 M_{\odot}$ , by using published models due to Iben (1967) and Kelsall and Strömgren (1969) and by assuming a Hyades age of log  $\tau$  (years) = 8.7. If log  $\tau$  (years) = 8.9, then  $M^* = 2.0-2.4 M_{\odot}$ .<sup>2</sup> It is probably only coincidental that the maximum mass for the existence of a helium flash in the core lies somewhere between 2.25 and 3  $M_{\odot}$  (for  $X_e = 0.71$ ) according to Iben (1967).

We now make a speculative suggestion concerning the parent stars which give rise to each white-dwarf sequence. Our suggestion is based primarily on the observational evidence from the Hyades Cluster with supporting observational evidence from other sources to be summarized below. It appears that stars with initial masses in the ranges 0.8  $M_{\odot}$  to  $M^*$ ,  $M^*$  to  $M^{\dagger}$ , and  $M^{\dagger}$  to an unknown limit, end up as white dwarfs with high mass ( $\sim 0.4-1 M_{\odot}$ ), very low mass ( $\sim 0.1 M_{\odot}$ ), and intermediate to high mass (up to  $\sim 1 M_{\odot}$ ), respectively. It also appears that  $M^* \approx 2.4 M_{\odot}$  and  $M^{\dagger} \geq 3 M_{\odot}$ . The supporting observational evidence is now summarized.

1. Since the initial stellar luminosity function falls off very rapidly with increasing mass, most of the evolved Hyades will have had initial masses actually very close to  $M_{\text{RG}}$ . It follows that evolution in the Hyades Cluster is producing mainly white dwarfs of  $\sim 1$  and  $\sim 0.1 M_{\odot}$  since, at the present epoch,  $M_{\text{RG}}$  is only slightly less than  $M^*$ .

2. A slow evolutionary phase of approximately  $2 \times 10^8$  years is required for the Hyades with initial  $M \approx M^*$  just before they cool down the red subluminoous sequence, in order to provide a source which is contemporary with the present red-giant source of the blue subluminoous sequence. This implies the expected presence of approximately two

<sup>2</sup> These ideas were originally advanced in a preliminary form by Stothers (1966, 1969), who attempted to use the two Eggen-Greenstein sequences of white dwarfs to infer  $M^*$ . Recently, Jones (1970) has independently made the same attempt (using the Eggen-Greenstein sequences) and has derived  $M^* \approx 1.9 M_{\odot}$ , but he has uncovered an apparent discrepancy with the Sirius group, which has nearly the same age as the Hyades. Our fundamentally different resolution of the white-dwarf sequences now removes this discrepancy (see text).

very bright red subdwarfs, which, as noted above, are actually observed and may be in a residual shell-burning phase.

3. The one possible white-dwarf member of the Pleiades (Eggen and Greenstein 1965), where evolved stars must have had initial masses greater than  $5\text{--}7\ M_{\odot}$ , has a present mass of about  $1\ M_{\odot}$  (Stothers 1970 and Figure 2).

4. The four blue white dwarfs which are possible members of the Sirius group (Jones 1970) have masses of  $1.0 \pm 0.1\ M_{\odot}$  if we apply Van Horn's procedure to reduce the observational data. These masses are in excellent agreement with the orbital mass of the brightest object, Sirius B, which is  $1.0\ M_{\odot}$  (Schwarzschild 1958). The normalized luminosity function of the four possible members resembles closely that of the blue white dwarfs in the Hyades. Since the Sirius group is very nearly of the same age as the Hyades (Eggen 1960), the present results for this stellar group may lend credence to the corresponding results for the Hyades. Praesepe, another cluster of similar age, has three known blue white dwarfs (Eggen and Greenstein 1965), all of similar mass (approximately  $1\ M_{\odot}$ ).

5. Most of the white dwarfs with high space velocity have large photometrically inferred masses, e.g., those in the  $\gamma$  Leonis, 61 Cygni, and W219 moving groups (Eggen 1959; Eggen and Greenstein 1965). Their high space velocities and the otherwise known old age of the  $\gamma$  Leonis and 61 Cygni groups imply that they originated primarily from low-mass main-sequence stars ( $\sim 1\ M_{\odot}$ ), which therefore seem to end up as white dwarfs of high mass, as we have suggested.

6. The central cores of *all* the white dwarfs are expected to be composed of carbon and oxygen or heavier elements if the helium-burning phase has been completed. This latter supposition is indicated at least for stars with initial masses of  $\sim 1$  and  $\sim 5\ M_{\odot}$ , according to the straightforward interpretation of the H-R diagrams of applicable clusters (see, e.g., Hayashi *et al.* 1962). For stars of  $\sim 2\ M_{\odot}$ , like those evolving in the Hyades, we have available the frequencies and luminosities of red giants. There seems to be little doubt that the four red giants in the Hyades are burning core helium, on the basis of a simple comparison with the numbers of evolving main-sequence stars (Schlesinger 1969) or of white dwarfs (this paper)—which yield a red-giant lifetime of  $(2\text{--}4) \times 10^8$  years—and with relevant stellar models due to Iben (1965, 1967) and to Cox and Salpeter (1964). Furthermore, Hubbard and Wagner (1970) have recently summarized the available evidence concerning the interior composition of white dwarfs as inferred from their present luminosities, masses, and radii. The only relatively clear-cut case is for 40 Eri B ( $0.4\ M_{\odot}$ ), whose interior probably has a carbon-oxygen, or heavier, chemical composition. Finally, we mention the direct observation of a carbon-rich atmosphere in a few high-velocity white dwarfs of probably high mass (Eggen and Greenstein 1965). However, it is *normally* to be expected that a thin hydrogen-helium envelope will remain with the star since mass loss becomes less effective as the mass of the envelope shrinks (Hayashi *et al.* 1962; Forbes 1968; Kippenhahn *et al.* 1968).

The red subluminous stars in the general field seem to belong mostly to the faint (high-mass) sequence of Eggen and Greenstein, as would be expected for the majority of old white dwarfs regardless of their space velocity. Among the possible exceptions with  $U - V > 0.0$ , Eggen's most accurate data (Eggen 1969, Fig. 1) show only four stars, all members of common-proper-motion pairs whose distances have been determined from spectroscopic parallaxes of the presumed companions. Three of these stars turn out to have moderately low tangential velocities and therefore could have originated recently from stars near the middle of the main sequence, while the fourth has a somewhat higher tangential velocity. However, the latter object is very red and is possibly a main-sequence star. Eggen's less accurate data (Eggen 1969, Fig. 2; Eggen 1970, Table 2) contain five bright subluminous stars with  $U - V > 0.0$ ; distances for all five have been based on trigonometric parallaxes. Since only two of these stars appear to have low tangential velocities, it is difficult to understand the origin of the other three

in the framework of our present discussion. There is a remote possibility that these three stars lie in the high-velocity tail of the intermediate Population I velocity distribution and have been preferentially detected because of their correspondingly large proper motions. However, since uncertainty subsists in their location on the H-R diagram, further observations are essential to establish whether these stars are possibly composite objects, as well as to verify the trigonometric parallaxes and proper motions for them. An independent method of locating these stars on the H-R diagram, which would apply also to the Hyades subluminescent stars, would be through spectroscopic or multi-colorimetric determination of their surface gravities. If they are intrinsically very bright and of low mass, then their surface gravities should be approximately solar. Decreasing brightness would be reflected in higher surface gravity, becoming as high as  $\sim 100 g_{\odot}$ . However, it should be kept in mind that if they are actually background main-sequence stars or foreground high-mass white dwarfs, the surface gravities would also be  $\sim 1 g_{\odot}$  and  $\sim 100 g_{\odot}$ , respectively.

In the future, if the red subluminescent stars in the Hyades and general field should turn out to be background main-sequence stars and/or foreground high-mass white dwarfs, then it would appear that parent stars with masses greater than  $\sim 2.4 M_{\odot}$  have a still obscure terminal evolution. However, our supporting arguments enumerated above (except for the first two) retain their relevance in this case.

In conclusion, we have interpreted the color-magnitude diagram for faint subluminescent stars in the Hyades in terms of two sequences containing most of the subluminescent stars: (1) white dwarfs with  $\sim 1 M_{\odot}$ , and (2) white dwarfs with  $\sim 0.1 M_{\odot}$ . It is hoped that further observational and theoretical work will produce more definitive information on the origin and evolution of subluminescent stars in a variety of star clusters.

It is a pleasure to thank Dr. I. Iben, Jr., for the loan of his computer subroutine to calculate stellar surface conditions and Dr. S. C. Vila for the communication of several unpublished model sequences of white dwarfs. Dr. P. Demarque kindly drew our attention to Kaminisi's work, and a referee made a number of important suggestions on the manuscript. One of us (Chin) gratefully acknowledges a faculty research fellowship for the summer of 1969, administered through the Research Foundation of the State University of New York.

#### REFERENCES

- Öhlm, K.-H. 1968, *Ap. and Space Sci.*, **2**, 375.  
 Chandrasekhar, S. 1939, *An Introduction to the Study of Stellar Structure* (Chicago: University of Chicago Press).  
 Cox, A. N., and Stewart, J. N. 1965, *Ap. J. Suppl.*, **11**, 22.  
 Cox, J. P., and Salpeter, E. E. 1964, *Ap. J.*, **140**, 485.  
 Eggen, O. J. 1959, *Observatory*, **79**, 135.  
 ———. 1960, *M.N.R.A.S.*, **120**, 563.  
 ———. 1968, *Ap. J.*, **153**, 195.  
 ———. 1969, *ibid.*, **157**, 287.  
 ———. 1970, *ibid.*, **159**, 945.  
 Eggen, O. J., and Greenstein, J. L. 1965, *Ap. J.*, **141**, 83.  
 Ezer, D., and Cameron, A. G. W. 1967, *Canadian J. Physics*, **45**, 3461.  
 Forbes, J. E. 1968, *Ap. J.*, **153**, 495.  
 Greenstein, J. L. 1969, *Comments on Astrophysics and Space Physics*, **1**, 62.  
 Hamada, T., and Salpeter, E. E. 1961, *Ap. J.*, **134**, 683.  
 Hayashi, C., Hōshi, R., and Sugimoto, D. 1962, *Progr. Theoret. Phys. Suppl.* (Kyoto), No. 22.  
 Hubbard, W. B., and Lampe, M. 1968, *Ap. J. Suppl.*, **18**, 297.  
 Hubbard, W. B., and Wagner, R. L. 1970, *Ap. J.*, **159**, 93.  
 Iben, I., Jr. 1963, *Ap. J.*, **138**, 452.  
 ———. 1965, *ibid.*, **142**, 1447.  
 ———. 1967, *ibid.*, **147**, 650.  
 Johnson, H. L. 1966, *Ann. Rev. Astr. and Ap.*, **4**, 193.  
 Jones, E. M. 1970, *Ap. J.*, **159**, 101.  
 Kaminisi, K. 1956, *Kumamoto J. Sci.*, **2**, 295.

- Kelsall, T., and Strömgren, B. 1969, *Stellar Astronomy*, ed H.-Y. Chiu, R. Warasila, and J. Remo (Gordon and Breach Publ.), Vol. 1, p. 237.
- Kippenhahn, R., Thomas, H.-C., and Weigert, A. 1968, *Zs. f. A p.*, **69**, 265
- Mestel, L., and Ruderman, M. A. 1967, *M N R. A S* , **136**, 27.
- Osterbrock, D. E. 1953, *A p. J.*, **118**, 529.
- Pesch, P. 1968, *A p. J.*, **151**, 605.
- Salpeter, E. E. 1961, *A p. J.*, **134**, 669.
- Sandage, A. 1957, *A p. J.*, **125**, 422.
- Schatzman, E. 1958, *White Dwarfs* (Amsterdam: North-Holland Publ )
- Schlesinger, B. M. 1969, *A p. J* , **157**, 533
- Schwarzschild, M. 1958, *Structure and Evolution of the Stars* (Princeton: Princeton University Press).
- Stothers, R. 1966, *A J.*, **71**, 943.
- . 1969, *Stellar Astronomy*, ed. H.-Y Chiu, R. Warasila, and J. Remo (New York: Gordon and Breach Publ.), Vol. 2, p. 205.
- . 1970, *Phys. Rev Letters*, **24**, 538
- van Altena, W. F. 1966, *A J.*, **71**, 482
- . 1969, *ibid.*, **74**, 2.
- van den Heuvel, E. P. J. 1970, *Pub. A S. P* , **81**, 815.
- Van Horn, H. M. 1968, *A p. J.*, **151**, 227.
- . 1970, *A p. J. (Letters)*, **160**, L53.
- Vila, S. C. 1966, *A p. J.*, **146**, 437.
- . 1967, *ibid.*, **149**, 613.
- . 1969, private communication

